

Exact G-functions for various leaf normal distribution functions applied to analytical two-stream radiative transport in turbid vegetation media

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Leaf normal distribution functions (LNDFs)

- Turbid media approach (Marshak and Davis, 2005): $g_L(\mathbf{x}, \mathbf{y}_L, t) \geq 0$ is the probability that a planar leaf at point \mathbf{x} and at time t , its normal vector has the direction \mathbf{y}_L is confined to the upper hemisphere S_1^+
- Parameterisation of S_1^+ as a function of the azimuth angle φ and the cosine μ of the zenith angle ϑ :

$$L_o \begin{pmatrix} \cos \varphi \sqrt{1 - \mu^2} \\ \sin \varphi \sqrt{1 - \mu^2} \\ \mu \end{pmatrix} := \hat{\omega}(\mu, \varphi)$$

with the unit length L_o and the parameter $(\mu, \varphi) \in [0, 1] \times [0, 2\pi]$

- Homogeneous and time independent LNDF with separated angular dependence $g_L(\mathbf{y}_L = \hat{\omega}(\mu_L, \varphi_L)) = g_\mu(\mu_L) g_\varphi(\varphi_L)$ with $g_\varphi(\varphi_L) := 1$
- Normalisation condition

$$\int_{[0,1]} g_\mu(\mu_L) d\mu_L = 1$$

- Standard LNDFs which fulfil the above normalisation condition: purely horizontal, erectophile, extremophile, uniform, plagiophile, planophile, rather vertical and purely vertical leaves normals (Otto and Trautmann, 2008a)

Explicit analytical G-functions (GFs)

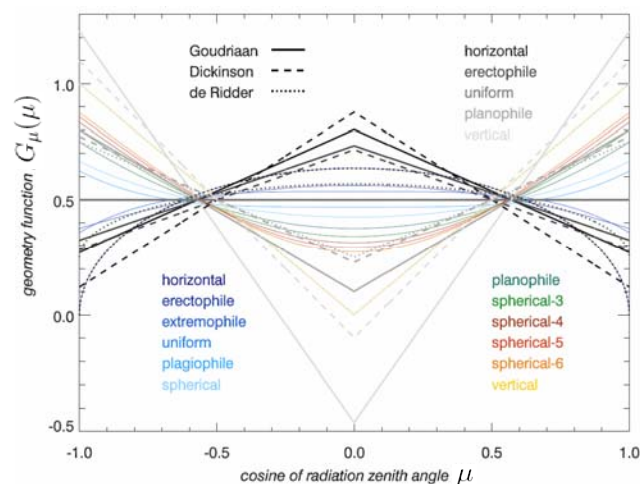
- Ross-Nilson intersection function is a geometry function which is calculated for turbid media as follows (Ross, 1981)

$$G(\mathbf{y}) := \frac{1}{A(S_1^+)} \int_{S_1^+} |\mathbf{e}_y \cdot \mathbf{e}_{y_L}| g_L(\mathbf{y}_L) d\mathbf{o}(\mathbf{y}_L) \quad \text{with } \mathbf{e}_z := \frac{\mathbf{z}}{\|\mathbf{z}\|}, \mathbf{z} \in \mathbb{R}^3$$

and is azimuthally independent and axially symmetric for standard LNDFs

$$G_\mu(\mu) := G(\mathbf{y} = \hat{\omega}(\mu, \varphi)) \quad \text{and} \quad G_\mu(\mu) \equiv G_\mu(-\mu)$$

- Explicit analytical expressions for standard LNDFs (Otto and Trautmann, 2008a) in comparison to approximations (Goudriaan, 1977; Dickinson et al., 1990; de Ridder, 1997):



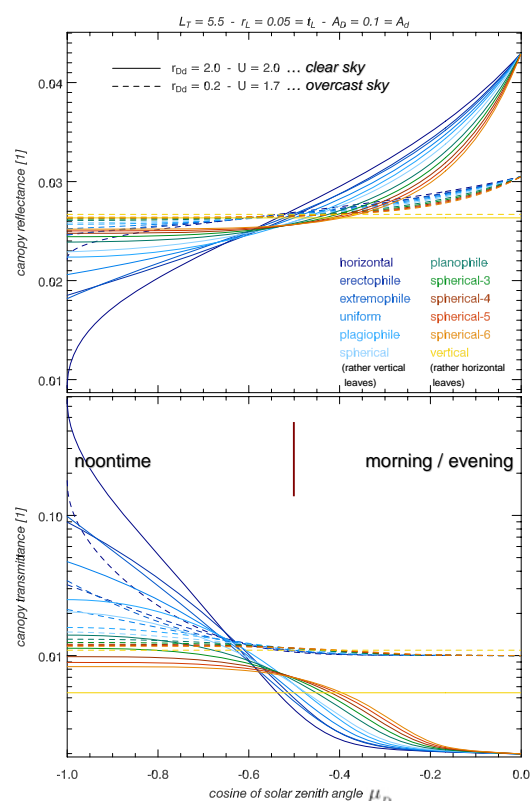
Two-stream approximation

- Time independent and horizontally homogeneous radiative transfer equation for turbid vegetation media assuming bi-Lambertian, elastic leaf scattering
- Two-stream approach for the upward and downward diffuse irradiances $E_\pm(L, \lambda)$ where the variable L is the leaf area index and λ the wavelength
- Analytical solution of the two-stream system of equations for standard LNDFs

$$\frac{d}{dL} \mathbf{E}(L, \lambda) = \begin{pmatrix} \alpha_1(\lambda) & -\alpha_2(\lambda) \\ \alpha_2(\lambda) & -\alpha_1(\lambda) \end{pmatrix} \cdot \mathbf{E}(L, \lambda) + \mathbf{D}(L, \lambda) \begin{pmatrix} -\alpha_3(\lambda) \\ \alpha_4(\lambda) \end{pmatrix}$$

(Otto and Trautmann, 2008b) for a homogeneous layer using exact GFs

- Input parameters: surface albedo of direct A_D and diffuse light A_d total leaf area index L_T , cosine solar zenith angle $\mu_D < 0$, diffusivity factor U , hemispherical leaf reflectance and transmittance r_L and t_L , downwelling direct and diffuse irradiances at the top of the vegetation layer E_D and E_d
- Simulation of the canopy reflectance (CR) and transmittance (CT) as a function of the cosine of the solar zenith angle for two ratios of incident direct and diffuse light $r_{Dd} = E_D / E_d$ in the UV-VIS spectral range:



Radiation transport depends significantly on the orientation of the foliage (LNDF) as well as on the sky conditions above the vegetation:

- + CR for overcast skies is larger/lower at noon/morning-evening than for clear skies
- + CT of rather horizontal foliage (deciduous forests) is always larger under overcast sky conditions
- The one-layer solution can be extended to vertically inhomogeneous medium

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